(6 pages)

Reg. No. : ....

Code No. : 20585 E **Sub. Code : SEMA 5 C**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2021.

Fifth Semester

Mathematics

Major Elective — COMBINATORIAL MATHEMATICS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. nC_0 = ___________.

   (a) 0 (b) 1

   (c) n (d) none

2. P(5) = ___________.

   (a) 5 (b) 20

   (c) 100 (d) 120

3. The number of different pairings of $2n$ objects is

(a) $(2n)!$ (b) $\frac{(2n)!}{n!}$

(c) $\frac{(2n)!}{(2!)^n n!}$ (d) $\frac{(2n)!}{2!}$

4. The system of distinct representatives for the sets $A_1 = \{1, 2\}$, $A_2 = \{4\}$, $A_3 = \{1, 3\}$, $A_4 = \{2, 3, 4\}$ is

(a) {1, 2, 3, 4} (b) {1, 4, 3, 2}

(c) {1, 3, 2, 4} (d) {1, 3, 4, 2}

5. In a simple tree, the degree of each vertex is

(a) ≤ 3 (b) $= 3$

(c) ≥ 3 (d) 1

6. The number of derangements of the 3 symbols 1, 2, 3 is

(a) 1 (b) 2

(c) 3 (d) 4

7. How many ways are there of placing 5 non-taking rooks on a 5×5 chess board?

(a) 3! (b) 4!

(c) 5! (d) 6!

8. For any board C, $r_1(C)$ = __________.

(a) number of rows of C

(b) number of squares of C

(c) number of columns of C

(d) none

9. In a block diagram, the subsets of S are called __________.

(a) blocks (b) varieties

(c) elements (d) none

10. In a block diagram each element lies in exactly __________ blocks?

(a) k (b) λ

(c) r (d) v

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) How many permutations are there of the 26 letters of the alphabet in which the five vowels are in consecutive places?

Or

(b) P.T. $\binom{n}{r} = \frac{n!}{r!(n-r)!}$.

12. (a) A pack of 52 cards is divided among 4 people so that each gets 13 cards. How many such deals are possible?

Or

(b) Define Latin square and give an example.

13. (a) If $a_n = 5a_{n-1} - 6a_{n-2}\ \forall n \geq 3$ and $a_1 = a_2 = 1$, find a_n.

Or

(b) Let $b_n = \binom{n}{0} + \binom{n-1}{1} + \binom{n-2}{2} + \ldots.$ Verify that $b_1 = 1$, $b_2 = 2$ and show that $b_n = b_{n-1} + b_{n-2}$ $\forall n \geq 3$.

14. (a) How many integers from 1 to 1000 are divisible by none of 3, 7, 11?

Or

(b) Find the rook polynomial of the board.

15. (a) Show that there exist no (12, 8, 3, 2, 1) configuration.

Or

(b) Show that there are no integers a, b, c such that $a^2 + b^2 = 6c^2$ apart from $a = b = c = 0$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$.

Or

(b) By using the identify $(1+x)^{2n} = (1+x)^n (1+x)^n$ and considering the coefficient of x^n on both sides, prove that $\binom{2n}{n} = \binom{n}{0}^2 + \binom{n}{1}^2 + \ldots + \binom{n}{n}^2$.

17. (a) Show that if a graph has $2n$ vertices, each of degree $\geq n$, then the graph has a perfect matching.

Or

(b) State and prove marriage theorem.

18. (a) If a_n denote the number of derangement on 1, 2, 3, ... n, prove that
$a_n = (n-1)[a_{n-1} + a_{n-2}]\ \forall n \geq 3$ and $a_1 = 0$, $a_2 = 1$.

Or

(b) Solve the recurrence relation for the Fibonacci sequence.

19. (a) If A and B are non interfering parts of a chess board C, prove that $R(x, C) = R(x, A) + R(x, B)$.

Or

(b) In constructing a 6×6 Latin square the first two rows have been chosen as follows :

1	2	3	4	5	6
2	4	1	3	6	5

In how many ways can a third row be chosen?

20. (a) Prove that for a (b, v, r, k, λ) configuration, $b \geq v$.

Or

(b) Prove that there is no finite projective plane of order 6.